Int. J. Heat Mass Transfer. Vol. 33, No. 6, pp. 1367–1370, 1990
Printed in Great Britain

0017-9310/90 $3.00+0.00

Heat transfer in liquid metals by natural convection

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(*Received* 3 *January* 1989 *and in final form* 14 *September* 1989)

INTRODUCTION

NATURAL CONVECTION in horizontal enclosures with heating from below has been the subject of many numerical studies in recent years. However, little attention has been paid to fluids having very low Prandtl numbers, such as liquid metals [1–4]. McDonough and Catton [1] studied natural convection using a mixed finite difference–Galerkin procedure in a horizontal square box which was heated from below and cooled from above, with the side walls insulated. They found that the numerical results with lower Pr did not converge as quickly as those with higher Pr, and that the convergence was not monotonic. They believed that this was due to the increasing nonlinearity as the Prandtl number decreased. In the numerical results presented in ref. [1], the Prandtl numbers were not lower than $Pr = 0.71$. Obviously, further numerical studies in the low Prandtl number range are needed, which is one of the objectives of this paper.

For pool boiling and two-phase flow heat transfer, the temperature distribution in liquid metals is crucial to the operation and boiling incipience. In a gravitational environment, heat transfer in liquid metals prior to boiling incipience is a problem of natural convection combined with conduction, even for liquid metal layers with a thickness of the order of 10 mm. Because of the limitations of experiments, a numerical study is needed to obtain temperature distributions prior to boiling and to study mechanisms of boiling incipience on the wall for very low Prandtl numbers. This is another motivation of the present numerical study.

Many pool boiling and evaporation test sections for liquid metal layers can be modeled as a two-dimensional partial heating configuration as shown in Fig. 1, with the dimension perpendicular to the paper being infinitely long. A uniform heat flux along the heating element is specified. The upper liquid surface is kept at the saturation temperature $T_s = T_c$, corresponding to the system pressure, and is considered as a free surface in contrast with the rigid lower surface. The container wall, except the heating element, is insulated to prevent heat loss to the environment. Another alternative boundary condition for this problem is to specify a constant temperature $T_h > T_s$ at the lower surface, which has been used more often in the previous numerical studies.

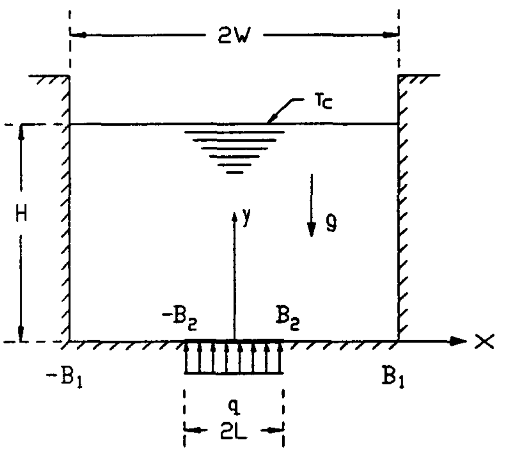

FIG. 1. Two-dimensional horizontal liquid metal layer with localized heating from below.

In this paper, the natural convection of fluids having very low Prandtl numbers down to $Pr = 0.00125$ in enclosures with partial or full heating from below will be studied numerically, and the numerical results will be compared with the existing experimental data [5] and the empirical heat transfer equation [2], respectively.

FORMULATION AND NUMERICAL METHOD

The dimensionless governing equations for steady two-dimensional laminar flow of a Newtonian fluid with no dissipation under the Boussinesq approximation can be written as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{\partial (U^2)}{\partial X}+\frac{\partial (UV)}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$\frac{\partial (UV)}{\partial X}+\frac{\partial (V^2)}{\partial Y}=-\frac{\partial P}{\partial Y}+Ra_1\,Pr\,\theta+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \tag{3}$$

$$\frac{\partial (U\theta)}{\partial X}+\frac{\partial (V\theta)}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

with the following dimensionless variables:

$$X=\frac{x}{H};\quad Y=\frac{y}{H};\quad U=u\frac{H}{\alpha};\quad V=v\frac{H}{\alpha};\quad \theta=\frac{T-T_c}{T_c};$$

$$P=(p+\rho_0 gy)H^2/\rho\alpha^2;\quad Pr=\nu/\alpha;\quad Ra_1=g\beta H^3 T_c/\alpha\nu$$

where

$$\beta=-\frac{1}{\rho_0}(\rho-\rho_0)/(T-T_0).$$

For the partial heating from below, two geometrical parameters are needed: $B_1 = W/H$ and $B_2 = L/H$.

The boundary conditions for the constant heat flux and free upper surface shown in Fig. 1 are

$$V=0,\ \frac{\partial U}{\partial Y}=0, \theta=0 \quad Y=1,\quad -B_1 \leqslant X \leqslant B_1 \tag{5a}$$

$$V=U=0,\ \frac{\partial \theta}{\partial X}=0 \quad 0 \leqslant Y \leqslant 1,\quad X=-B_1 \tag{5b}$$

$$V=U=0,\ \frac{\partial \theta}{\partial X}=0 \quad 0 \leqslant Y \leqslant 1,\quad X=B_1. \tag{5c}$$

For the rigid lower boundary condition ($Y = 0$)

$$V=U=0 \quad -B_1 \leqslant X \leqslant B_1 \tag{5d}$$

NOMENCLATURE

B_1	W/H
B_2	L/H
g	gravitational acceleration [m s^{-2}]
H	height of thermal cavities or height of the liquid level [m]
k	thermal conductivity [W m^{-1} °C^{-1}]
L	half width of the heating element [m]
Nu	Nusselt number, $qH/k(T_h - T_c)$
p	pressure [N m^{-2}]
P	$(p+\rho_0 g y)H^2/\rho\alpha^2$
Pr	Prandtl number
q	heat flux [W m^{-2}]
Ra	Rayleigh number, $g\beta H^3(T_h - T_c)/\nu\alpha$
Ra_1	Rayleigh number, $g\beta H^3 T_c/\nu\alpha$
T	temperature [°C]
T_0	reference temperature [°C]
T_s	saturation temperature [°C]
T_h	hot surface temperature [°C]
T_c	cold surface temperature [°C]
T_w	temperature of heating surface in the case of the constant heat flux [°C]
U, V	dimensionless velocities, uH/α, vH/α
u, v	velocities [m s^{-1}]
W	width of the thermal cavity [m]
X, Y	dimensionless coordinate directions, x/H, y/H
x, y	coordinate directions.

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
β	coefficient of volumetric thermal expansion [°C^{-1}]
θ	dimensionless temperature, $(T-T_c)/(T_h-T_c)$ or $(T-T_c)/T_c$
ν	kinematic viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
ρ_0	reference density [kg m^{-3}].

$$\frac{\partial\theta}{\partial Y} = -\frac{qH}{T_c k} \quad -B_2 \leqslant X \leqslant B_2 \tag{5e}$$

$$\frac{\partial\theta}{\partial Y} = 0 \quad -B_1 \leqslant X < -B_2 \quad \text{and} \quad B_2 < X \leqslant B_1. \tag{5f}$$

For the case of a constant temperature T_h at the lower surface, the dimensionless numbers θ and Ra_1 need to be changed to

$$\theta = \frac{T-T_c}{T_h-T_c} \quad \text{and} \quad Ra = g\beta H^3(T_h-T_c)/\alpha\nu. \tag{5g}$$

The corresponding boundary condition (5e) is replaced by

$$\theta = 1 \quad -B_2 \leqslant X \leqslant B_2. \tag{5h}$$

The problem is specified mathematically by equations (1)–(5). The solution procedure used for solving these equations is the finite-difference method SIMPLER, which has been described in detail by Patankar [6, 7]. The power-law scheme is used for the convection–diffusion terms and the discretization equations are solved by using the tridiagonal matrix algorithm (TDMA or Thomas algorithm). Underrelaxation parameters are used to control the advancement of the solutions and to ensure convergence. The computer program was tested with different grid sizes for the same problem, and the solutions proved to be essentially independent of the grid size.

RESULTS AND DISCUSSION

1. *Partial heating with constant heat flux*

The numerical calculations were conducted with the configuration in Fig. 1, and the results of the temperature distribution at $x = 0$ were compared with the experimental data from Takenaka *et al.* [5]. The experiment was made with a horizontal potassium layer heated from below. The test vessel had a rectangular cross-section of 140×96 mm^2. The effective heating area in the center of the vessel is 100×20 mm^2. The vertical liquid temperature distribution was measured by sliding thermocouples along the central line of the vessel. Since the length of the heating element is much larger than its width, the heat transfer can be modeled as two-dimensional natural convection within the configuration as shown in Fig. 1, with $B_1 = 1.70$, $B_2 = 0.360$, $q = 10^5$ W m^{-2} and $T_c = T_s = 527$°C. The reference temperature T_0 is chosen as T_s and the relevant properties can be found from ref. [8], thus giving $Pr = 0.004$ and $Ra_1 = 3.0 \times 10^6$. Figure 2 shows the comparison of the numerical temperature distribution at $x = 0$ and the experimental data of $H = 28$ mm from ref. [5]. It can be seen that the agreement is generally good and is excellent near the lower surface. The grid size used in the numerical calculation was 33×20. The temperature distribution consists of the boundary region near the heating surface and the bulk region. The boundary thickness is relatively larger compared with those of ordinary fluids. Figure 3 shows the corresponding temperature contours and dimensionless stream function contours. Because of the symmetry, only the solutions for the half cavity were presented in the figure. The isotherms and streamlines indicate that a plume-like flow is generated above the heated region, which is the normal case for natural convection in a liquid heated from below with a rigid or free upper surface [9, 10].

Figure 4 shows the temperature distributions for sodium with different values of the heat flux q. Since the temperature of the upper free surface is fixed at T_c, higher values of the heat flux result in a higher T_w. For example, the T_w of the case with $q = 30$ W cm^{-2}, is almost 50°C higher than that of the case with $q = 5$ W cm^{-2}. Also, for the low heat flux the temperature field shifts towards the conduction regime with an almost linear temperature distribution.

In the numerical calculations, it was found that the convergence speed is much slower with lower values of the

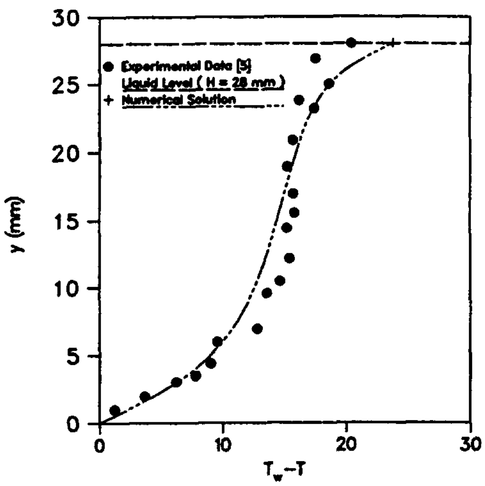

FIG. 2. Comparison between the numerical solution and the experimental data.

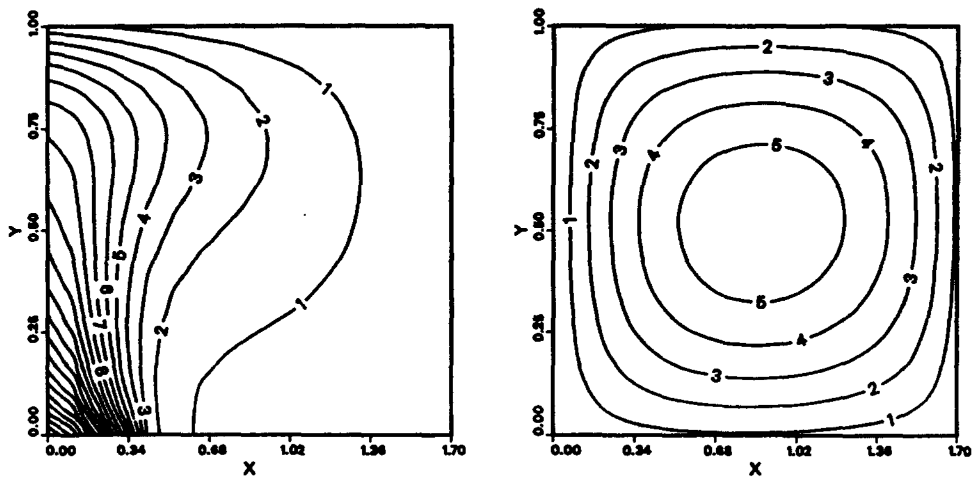

FIG. 3. Temperature and stream function contours for the half cavity.

Prandtl number, as indicated in refs. [1, 11]. Considering this fact, the calculations proceeded from higher to lower Prandtl numbers with the solutions of the higher Pr as the starting values for the lower Pr. Also, under-relaxation was needed to ensure convergent solutions. The under-relaxation parameter used in the above calculations was 0.1.

2. *Natural convection in a horizontal rigid cavity with specified boundary temperature*

The problem of natural convection in a rigid cavity with specified boundary temperatures will now be examined. The boundary conditions with reference to Fig. 1 in this case are changed to the following:

$$V = U = 0, \theta = 0 \quad Y = 1, \quad -B_1 \leqslant X \leqslant B_1$$

$$V = U = 0, \theta = 1 \quad Y = 0, \quad -B_1 \leqslant X \leqslant B_1$$

$$V = U = 0, \frac{\partial \theta}{\partial X} = 0 \quad 0 \leqslant Y \leqslant 1, \quad X = -B_1$$

$$V = U = 0, \frac{\partial \theta}{\partial X} = 0 \quad 0 \leqslant Y \leqslant 1, \quad X = B_1.$$

The dimensionless temperature and Rayleigh numbers are $\theta = (T-T_c)/(T_h-T_c)$ and $Ra = g\beta H^3(T_h-T_c)/\alpha\nu$, respectively.

Raithby and Hollands [2] have proposed an empirical equation for the natural convection heat transfer in horizontal cavities nonextensive in the horizontal direction, which is

$$Nu = 1+[1-Ra_c/Ra]^*[k_1+2(Ra^{1/3}/k_2)^{1-\ln(Ra^{1/3}/k_2)}] + \left[\left(\frac{Ra}{5380}\right)^{1/3}-1\right]^* (1-\exp\{-0.95[(Ra/Ra_c)^{1/3}-1]^*\}) \tag{6}$$

where Ra_c is the critical Rayleigh number, and k_1 and k_2 are both functions of Pr, given as follows:

$$k_1 = 1.44/(1+0.018/Pr+0.00136/Pr^2) \tag{7}$$

$$k_2 = 75\exp(1.5\,Pr^{-1/2}). \tag{8}$$

The square brackets with the asterisk, $[\,]^*$, indicate that only positive values of the argument are to be taken, i.e.

$$[X]^* = (|X|+X)/2. \tag{9}$$

The above equation has been tested against experimental data for gases and liquids of various Prandtl numbers except liquid metals, with a maximum difference of 25%.

It is of interest to compare the present numerical solutions with equation (6) and to fill the gap in the low Prandtl number range. The calculations were carried out with $B_1 = 0.5$ (i.e. a square box) and the results are presented in Fig. 5. It can be seen that the agreement between the numerical solutions and equation (6) is generally good. The maximum difference within the range of comparison is less than 20%. A grid size of 25×25 was used in the numerical calculations, and the solutions were obtained from higher to lower Pr with the solutions of the higher Pr as the initial guesses for the lower Pr. The number of iterations for convergence ranged from 700 to 1000.

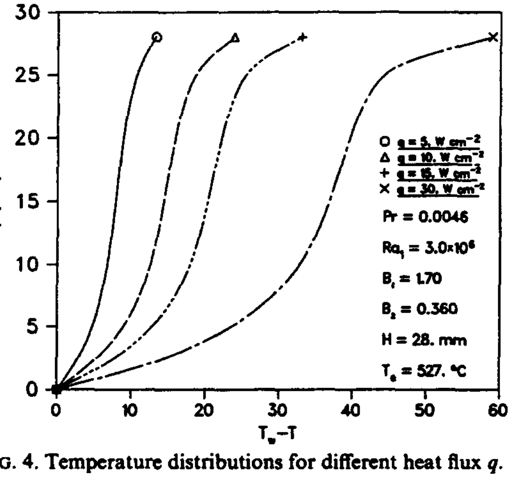

FIG. 4. Temperature distributions for different heat flux q.

CONCLUSIONS

A numerical study has been reported for natural convection in horizontal liquid metal layers with localized heating from below. The temperature distribution of the numerical results at $x = 0$ agrees well with the corresponding experimental data for a potassium layer. The isotherms and streamlines indicate that a plume-like flow is generated above the heated region. The numerical results based on a rigid square box with insulated side walls, T_h on the bottom and T_c on the top, generally agree with equation (6) of ref. [2], within the Rayleigh number range of $5 \times 10^3 - 10^5$, and the Prandtl number range of 0.00125–0.01.

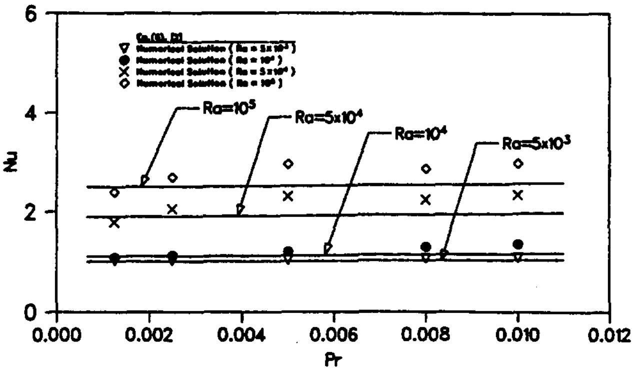

FIG. 5. Comparison of equation (6) and the present numerical solutions.

Acknowledgement—Funding for this work was provided by a joint effort of the Air Force and NASA under contract F33615-88-C-2820.

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Int. J. Heat Mass Transfer. Vol. 33, No. 6, pp. 1370–1373, 1990
Printed in Great Britain
0017-9310/90 \$3.00+0.00

Mixed convection experiments about a horizontal isothermal surface embedded in a water-saturated packed bed of spheres

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(*Received* 21 *April* 1989 *and in final form* 14 *September* 1989)

INTRODUCTION

NUMEROUS research articles have appeared in the literature in recent years in the area of convective heat transfer in fluid-saturated porous materials. The great majority of these articles deal with problems in natural convection or forced convection. The area of mixed convection, which constitutes the interface between natural and forced convection in porous media, has been, by comparison, largely overlooked.

One of the early investigations of combined free and forced